J.K. SHAH CLASSES

JKSC : 2018 – 19 | SYJC – COMMERCE

SYJC TEST SERIES | MATHEMATICS & STATISTICS

PAPER – II

TEST – 02

DURATION : 1HR 30 MIN | CORRELATION & REGRESSION | MARKS : 30

SOLUTION SET

Q1. ATTEMPT ANY 5 OUT OF 6 (2 MARKS EACH) (10)

01. coefficient of correlation between the variables X and Y is 0.3 and their covariance is 12 . The variance of X is 9 . Find standard deviation of Y

$r = 0.3$, $cov(x,y) = 12$, $\sigma x^2 = 9$,

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$0.3 = \frac{12}{3 \times \sigma y}$$

$$\sigma y = \frac{12}{3 \times 0.3} = 13.33$$

02. The coefficient of rank correlation for a certain group of data is 2/3 . If $\Sigma d^2=55$, assuming no ranks repeated ;find the no. of pairs of observation

$R = 2/3$; $\Sigma d^2 = 55$

$$\mathbf{R} = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$\frac{2}{3} = 1 - \frac{6(55)}{n(n^2 - 1)}$$

$$\frac{6(55)}{n(n^2 - 1)} = 1 - \frac{2}{3}$$

$$\frac{6(55)}{n(n^2 - 1)} = \frac{1}{3}$$

$$n(n^2 - 1) = 6 \times 55 \times 3$$

$$n(n^2 - 1) = 990$$

$$(n - 1).n.(n + 1) = 9.10.11$$

On comparing ; $n = 10$

03. for a bivariate data $b_{yx} = -1.2$ and $b_{xy} = -0.3$. Find correlation coefficient between x and y

SOLUTION

$r^2 = byx \times bxy$

$r^2 = -1.2 \times -0.3$

$r^2 = \frac{12}{10} \times \frac{3}{10}$

$r^2 = \frac{36}{100}$

$r = \pm \frac{6}{10}$

$r = -\frac{6}{10}$ (byx & bxy are –ve)

$r = -0.6$

04. The bivariate frequency distribution of weight (kg) and height of 60 students of SYJC as follows

Weight	Height (in cm)			
(in kg)	100 – 109	110 – 119	120 – 129	130 – 139
40 – 44	9	6	–	3
45 – 49	6	3	3	1
50 – 54	–	6	3	3
55 – 59	3	4	7	3

a) Find marginal frequency distribution of weight

b) Find conditional distribution of weight when height lies between 110 – 119

SOLUTION

MARGINAL FREQUENCY DISTRIBUTION OF WEIGHT

CI	40 – 44	45 – 49	50 – 54	55 – 59	TOTAL
F	18	13	12	17	60

CONDITIONAL DISTRIBUTION OF WEIGHT WHEN HEIGHT LIES IN 110 – 119

CI	40 – 44	45 – 49	50 – 54	55 – 59	TOTAL
F	6	3	6	4	19

05. if for bi – variate data $\bar{x} = 10$ and $\bar{y} = 12$, variance $V(x) = 9$; $\sigma y = 4$ & $r = 0.6$

Estimate x when y = 10

SOLUTION

X on Y

$$bxy = \frac{r\,\sigma x}{\sigma y} = 0.6 \times \frac{3}{4} = 0.45$$

$x - \bar{x} = bxy(y - \bar{y})$

$x - 10 = 0.45(y - 12)$

Put y = 10

$x - 10 = 0.45(10 - 12)$

$x - 10 = 0.45(-2)$

$x - 10 = -0.9$

$x = 9.1$

06. for 50 students of a class the regression equation of marks in Statistics (x) on the marks in accounts (y) is $3y - 5x + 180 = 0$. The mean of marks of accounts is 44 Find mean marks of Statistics

SOLUTION

X on Y : $3y - 5x + 180 = 0$

Put $\bar{y} = 44$

$3(44) - 5x + 180 = 0$

$132 - 5x + 180 = 0$

$312 - 5x = 0$

$5x = 312$

$\bar{x} = 62.4$

Q2. ATTEMPT ANY 4 OUT OF 5 (3 MARKS EACH) (12)

01. Compute rank correlation coefficient for the following data

Rx	:	1	2	3	4	5	6
Ry	:	6	3	2	1	4	5

SOLUTION

x	y	d = |x – y|	d^2
1	6	5	25
2	3	1	1
3	2	1	1
4	1	3	9
5	4	1	1
6	5	1	1
			$\Sigma d^2 = 38$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(38)}{6(36 - 1)}$$

$$= 1 - \frac{38}{35}$$

$$= -\frac{3}{35}$$

$$= -0.086$$

02. find number of pair of observations from the following data

$r = 0.4$; $\sum xy = 108$; $SDy = 3$; $\sum x^2 = 900$; where x and y are deviations from their respective means

SOLUTION

$r = 0.4$; $\Sigma(x - \overline{x})(y - \overline{y}) = 108$;
$\sigma y = 3$; $\Sigma(x - \overline{x})^2 = 900$

$$\sigma y = 3$$

$$\sqrt{\frac{\Sigma(y - \overline{y})^2}{n}} = 3$$

$$\sqrt{\Sigma(y - \overline{y})^2} = 3\sqrt{n}$$

$$r = \frac{\Sigma(x - \overline{x}).(y - \overline{y})}{\sqrt{\Sigma(x - \overline{x})^2}\sqrt{\Sigma(y - \overline{y})^2}}.$$

$$0.4 = \frac{108}{\sqrt{900} \,.\, 3\sqrt{n}}$$

$$\frac{4}{10} = \frac{108}{30 \,.\, 3\sqrt{n}}$$

$$\sqrt{n} = \frac{108 \text{ X } 10}{30 \text{ X } 3 \text{ X } 4}$$

$$\sqrt{n} = 3$$

Squaring ; $n = 9$

03. $n = 15$; $\bar{x} = 25$; $\bar{y} = 18$; $\sigma x = 3.01$; $\sigma y = 3.03$,

$\Sigma(x - \bar{x})(y - \bar{y}) = 122$. Find the correlation coefficient

SOLUTION

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$r = \frac{\frac{\Sigma(x - \bar{x})(y - \bar{y})}{n}}{\sigma x \cdot \sigma y}$$

$$r = \frac{\frac{122}{15}}{3.01 \times 3.03}$$

$$r = \frac{122}{15 \times 3.01 \times 3.03}$$

Taking log on both sides

$$\text{Log } r = \log 122 - \left(\log 15 + \log 3.01 + \log 3.03\right)$$

$$\text{Log } r = 2.0864 - \left(1.1761 + 0.4786 + 0.4814\right)$$

$$\text{Log } r = 2.0864 - 2.1361$$

$$\text{Log } r = \bar{1}.9503$$

$$r = AL\,(\bar{1}\,.\,9503)$$

$$r = 0.8919$$

04. $\sum x_i = 56$; $\sum y_i = 56$; $\sum x_i^2 = 476$; $\sum y_i^2 = 476$; $\sum x_i y_i = 469$, $n = 7$

a) Obtain linear regression of Y on X   b) y if x = 12

SOLUTION

$$\bar{x} = \frac{\Sigma x}{n} = \frac{56}{7} = 8$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{56}{7} = 8$$

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(469) - (56)(56)}{7(476) - (56)^2}$$

$$= \frac{3283 - 3136}{3332 - 3136}$$

$$= \frac{147}{196}$$

$$= \frac{21}{28}$$

$$= \frac{3}{4}$$

$$= 0.75$$

Equation

$$y - \bar{y} = byx\,(x - \bar{x})$$

$$y - 8 = 0.75(x - 8)$$

$$y - 8 = 0.75x - 6$$

$$y = 0.75x + 2$$

Put x = 12

$$y = 0.75(12) + 2$$

$$y = 9 + 2$$

$$y = 11$$

05. you are given below the following information about advertising and sales

	Adv. Exp. (x)	sales (y)	in lacs
Mean	10	90	
S.D.	3	12	r = 0.8 .

Obtain the regression line to estimate the likely sales when adv. budget is ☐ 15 lacs

SOLUTION

y on x

$$byx = r . \frac{\sigma y}{\sigma x}$$

$$= 0.8 \times \frac{12}{3}$$

$$= 0.8 \times 4 = 3.2$$

$y - \overline{y} = byx(x - \overline{x})$

$y - 90 = 3.2(x - 10)$

$y - 90 = 3.2x - 32$

$y = 3.2x - 32 + 90$

$y = 3.2x + 58$

Put $x = 15$

$y = 3.2(15) + 58$

$y = 48 + 58$

$y = 106$

Sales = ☐ 106 lacs when
Adv. budget = ☐ 15 lacs

Q3. ATTEMPT ANY 2 OUT OF 3 (4 MARKS EACH) (08)

01. the equ's of the regression lines are $2x + 3y - 6 = 0$ & $5x + 7y - 12 = 0$

Find a) correlation coefficient b) $\sigma x/\sigma y$

SOLUTION

STEP 1

ASSUME

XON Y : $5x + 7y - 12 = 0$

$5x = -7y + 12$

$x = \frac{-7y}{5} + \frac{12}{5}$

$bxy = -\frac{7}{5}$

Y ON X : $2x + 3y - 6 = 0$

$3y = -2x + 6$

$y = -\frac{2}{3}y + \frac{6}{3}$

$byx = \frac{-2}{3}$

STEP 2

$r^2 = bxy \cdot byx$

$= \frac{-7}{4} \times \frac{-2}{3}$

$= \frac{14}{15}$

Since $0 \leq r^2 \leq 1$

Our assumptions are correct

$r = \pm\sqrt{\frac{14}{15}}$

$r = -\sqrt{\frac{14}{15}}$ (byx & bxy are –ve)

$\log r' = \frac{1}{2}\left(\log 14 - \log 15\right)$

$\log r' = \frac{1}{2}\left(1.1461 - 1.1761\right)$

$\log r' = \frac{1.1461}{2} - \frac{1.1761}{2}$

$\log r' = 0.5730 - 0.5880$

$\log r' = \bar{1}.9850$

$r' = AL(\bar{1}.9850)$

$r' = 0.9661$

$r = -0.9661$

STEP 3

$$bxy = r \frac{\sigma x}{\sigma y}$$

$$\frac{-7}{5} = -0.9661 \times \frac{\sigma x}{\sigma y}$$

$$\frac{\sigma x}{\sigma y} = \frac{7}{5 \times 0.9661}$$

$$= \frac{7}{4.8304}$$

$$= 1.449$$

02. x : 3 2 1 5 4

y : 8 4 10 2 6

Compute Karl Pearsons Correlation coefficient

SOLUTION

x	y	$x-\overline{x}$	$y-\overline{y}$	$(x-\overline{x})^2$	$(y-\overline{y})^2$	$(x-\overline{x})(y-\overline{y})$
3	8	0	2	0	4	0
2	4	−1	−2	1	4	2
1	10	−2	4	4	16	−8
5	2	2	−4	4	16	−8
4	6	1	0	1	0	0
15 Σx	30 Σy	0 $\Sigma(x-\overline{x})$	0 $\Sigma(y-\overline{y})$	10 $\Sigma(x-\overline{x})^2$	40 $\Sigma(y-\overline{y})^2$	−14 $\Sigma(x-\overline{x})(y-\overline{y})$
$\overline{x} = 3$ $\overline{y} = 6$						

$$r = \frac{\Sigma(x-\overline{x}).(y-\overline{y})}{\sqrt{\Sigma(x-\overline{x})^2}\sqrt{\Sigma(y-\overline{y})^2}.}$$

$$r = \frac{-14}{\sqrt{10} \times \sqrt{40}}$$

$$r = \frac{-14}{\sqrt{400}}$$

$$r = \frac{-14}{20}$$

$$r = -0.7$$

03. for an experimental project , a Company collected data of 7 persons from Human resource development department referring to years of service and their monthly incomes

Years of service	:	11	07	09	05	08	06	10
Income (in 000's)	:	10	08	06	05	09	07	11

Find the regression equation of income on the years of service

SOLUTION

x	y	$x - \overline{x}$	$y - \overline{y}$	$(x - \overline{x})^2$	$(y - \overline{y})^2$	$(x - \overline{x})(y - \overline{y})$
11	10	3	2	9		6
7	8	– 1	0	1		0
9	6	1	– 2	1		– 2
5	5	– 3	– 3	9		9
8	9	0	1	0		0
6	7	–2	– 1	4		2
10	11	2	3	4		6
56	56	0	0	28		21
Σx $\overline{x} = 7$	Σy $\overline{y} = 7$			$\Sigma(x - \overline{x})^2$	$\Sigma(y - \overline{y})^2$	$\Sigma(x - \overline{x})(y - \overline{y})$

$$byx = \frac{\Sigma(x - \overline{x})(y - \overline{y})}{\Sigma(x - x)^2}$$

$$= \frac{21}{28}$$

$$= \frac{3}{4}$$

$$= 0.75$$

$$y - \overline{y} = byx\ (x - \overline{x})$$

$$y - 8 = 0.75(x - 8)$$

$$y - 8 = 0.75x - 6$$

$$y = 0.75x - 6 + 8$$

$$y = 0.75x + 2$$

Put x = 13

$$y = 0.75(13) + 2$$

$$= 9.75 + 2$$

$$= 11.75(000's)$$